WHAT TO DO IF YOU HAVE TOO LITTLE DATA?

Application of Bayesian approximate MI with longitudinal data

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TOO LITTLE DATA?

- Longitudinal studies often deal with small samples
- Measurement invariance (MI) over time is important:
  - (Wrongly) assuming invariance leads to biased estimates in latent growth models (LGMs).[1]
- Accurate estimation of partial invariance leads to reliable estimates[2]
  - Computationally challenging;
  - Misspecification introduces bias

[1] Leite, 2007

[2] Winter & Depaoli, in prep

SOLUTION?

- Bayesian approximate MI
  - Does not have same computational issues as frequentist estimators;
  - Can be used to accurately identify non-invariant parameters[3]

[3] Muthén & Asparouhov, 2013

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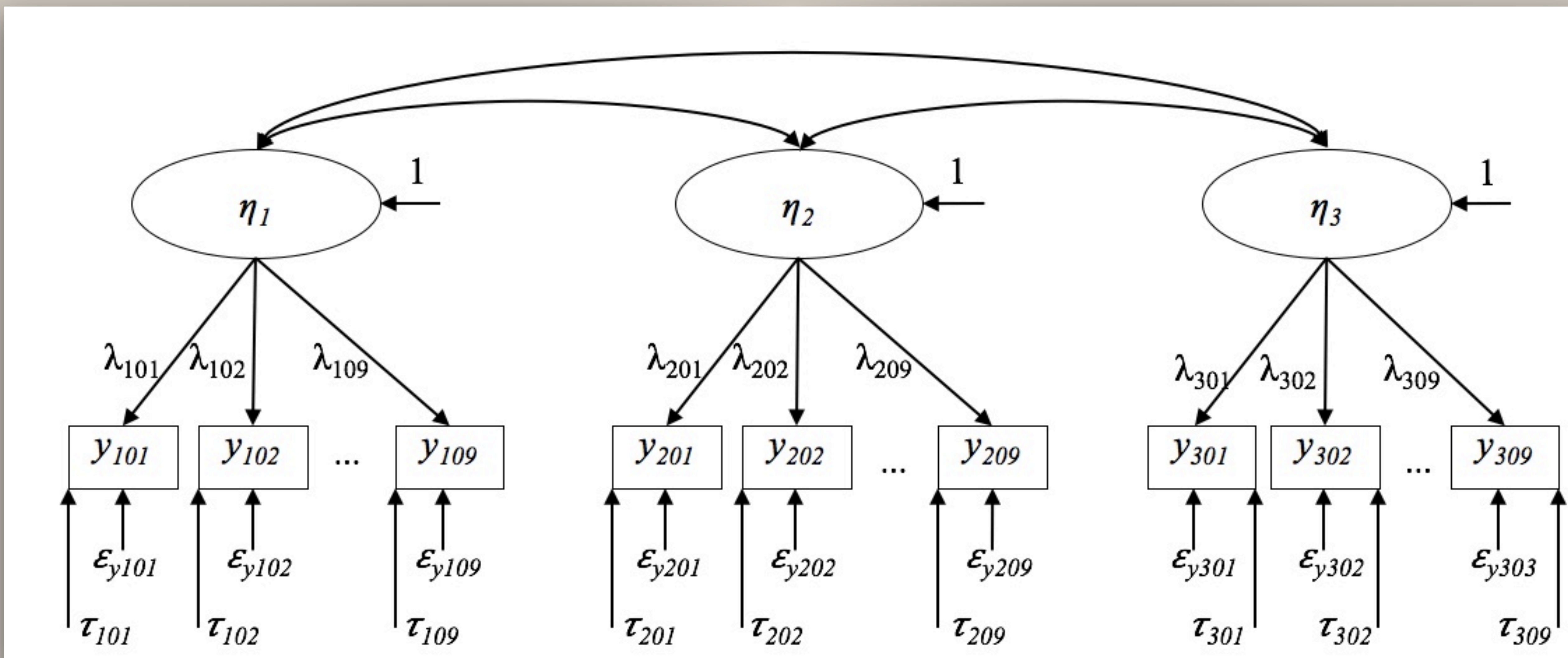
MODEL SPECIFICATION



CONVENTIONAL APPROACH

- Initially estimated a series of longitudinal CFAs using MLR to assess invariance:[6]

Model	χ^2	df	$\Delta\chi^2$
Configural	875.88	321	
Metric	903.24	337	18.50*ns*
Scalar	954.66	355	52.78*

* $p < .001$

[6] $\Delta\chi^2$ computed using scaling correction formula for MLR

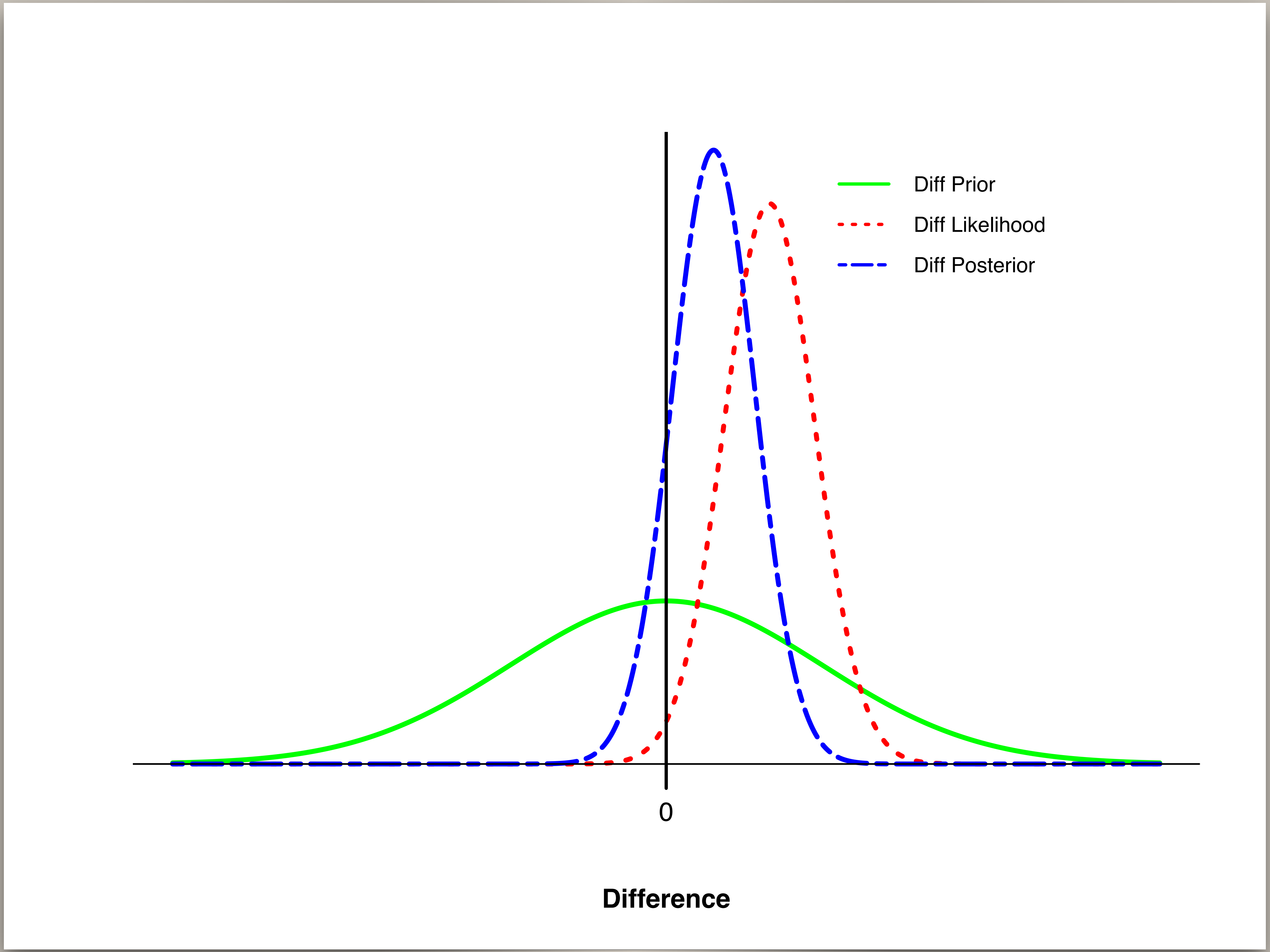
BAYESIAN APPROACH

- Specify small variance difference priors on the measurement parameters
- As implemented in *Mplus* [7]

```
MODEL:
          stress1 by y104* y105-y107 y113 y118-y121 (lam1_1-lam1_9);
          stress2 by y204* y205-y207 y213 y218-y221 (lam2_1-lam2_9);
          stress3 by y304* y305-y307 y313 y318-y321 (lam3_1-lam3_9);

          [y104-y107 y113 y118-y121] (tau1_1-tau1_9);
          [y204-y207 y213 y218-y221] (tau2_1-tau2_9);
          [y304-y307 y313 y318-y321] (tau3_1-tau3_9);

          stress1@1;
          stress2@1;
          stress3@1;
          [stress1-stress3@0];

MODEL PRIORS:
          DO(1,9) DIFF(lam1_#-lam3_#)~N(0,0.50);
          DO(1,9) DIFF(tau1_#-tau3_#)~N(0,0.50);
```

[7] L. Muthén & Muthén, 1998-2017

BAYESIAN APPROACH



BAYESIAN APPROACH

DIFFERENCE OUTPUT

	Average	Std. Dev.	Deviations from the Mean		
10	3.175	0.084	TAU1_1 0.242*	TAU2_1 -0.031	TAU3_1 -0.211*
11	2.756	0.093	TAU1_2 0.161*	TAU2_2 -0.084	TAU3_2 -0.078
12	2.181	0.092	TAU1_3 0.130*	TAU2_3 -0.063	TAU3_3 -0.067
13	2.773	0.076	TAU1_4 0.068	TAU2_4 0.069	TAU3_4 -0.136*
14	2.615	0.092	TAU1_5 0.150*	TAU2_5 -0.129	TAU3_5 -0.022
15	2.613	0.105	TAU1_6 0.111	TAU2_6 -0.053	TAU3_6 -0.058
16	2.536	0.097	TAU1_7 0.138	TAU2_7 -0.142	TAU3_7 0.004
17	2.225	0.106	TAU1_8 0.137	TAU2_8 -0.077	TAU3_8 -0.060
18	1.856	0.082	TAU1_9 0.153*	TAU2_9 0.030	TAU3_9 -0.185*

BAYESIAN APPROACH

- Ideally, you would free parameters with large deviations and estimate a Bayesian partial MI model
- For illustration sake, I wanted to show what happens if you use these results to estimate a partial MI model with MLR:

Model	χ^2	df	$\Delta\chi^2$
Configural	875.88	321	
Metric	903.24	337	18.50*ns*
Partial	920.86	347	14.29*ns*
Scalar	954.66	355	52.78*

* $p < .001$

CONCLUSION

- Bayesian approximate MI offers a pragmatic solution to problems often encountered with small sample longitudinal studies
  - Can be used to distinguish between small variations and large and meaningful differences between items over time
  - Can be used when running into computational difficulties

- Some current limitations
  - Support only for continuous and dichotomous items
  - Unclear how to decide the variance of the difference prior

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- Muthén, B. O., & Asparouhov, T. (2013). *BSEM Measurement Invariance Analysis. Mplus Web Notes: No. 17.* Retrieved from http://www.statmodel.com/examples/webnotes/webnote17.pdf

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WHAT TO DO IF YOU HAVE TOO MUCH DATA?

Application of Bayesian approximate m.i. in large cross-cultural comparisons

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Spoiler alert!

OPPOSITE PROBLEM

Too little ⟷ Too much

Amount of data

SAME SOLUTION

Bayesian approximate measurement invariance

TWO GOALS:

Making sure our latent construct means the same across countries, by….

1. ignoring **small** measurement artefacts whose effect on substantive conclusions is negligible

2. detecting **large** measurement artefacts that lead to erroneous substantive conclusions

GOAL 1: IGNORE SMALL DIFFERENCES

Classical approach

Exact zero constraints

↓

Frequent rejection of these constraints with many groups/time points

↓

Large series of model modifications that may capitalize on chance

↓

Mind boggling search through all possible combinations of measurement restrictions

with 20 countries, the number of possible combinations of restrictions run in the tens of millions

GOAL 1: IGNORE SMALL DIFFERENCES

Alternative: Bayesian approximate approach

Small differences automatically accounted for by the model [1]

```
MODEL:

%OVERALL%
f1 by y1* y2 y3 y4 (lam#_1-lam#_4);
 [y1-y4](nu#_1-nu#_4);

%G#1%
[f1@0];
 f1@1;

%G#2%
[f1];
 f1@1;

MODEL PRIOR:
DO(1,4) DIFF (lam1_#-lam2_#) ~ N(0,.01);
DO(1,4) DIFF (nu1_#-nu2_#) ~ N(0,.01);
```

1 Muthén & Asparouhov, 2013

SMALL SIMULATION STUDY[1]

True latent mean difference = .5

	PPP	estimate
unsystematic		
systematic		
large		

1 Lek et al., in press

SMALL SIMULATION STUDY[1]

True latent mean difference = .5

Every iteration, two LRTs

1. current model - original data
2. current model - newly generated data based on current model

PPP = P(2 > 1)

	PPP	estimate
unsystematic		
systematic		
large		

1 Lek et al., in press

SMALL SIMULATION STUDY[1]

True latent mean difference = .5

	PPP	estimate
unsystematic	.269	.477
systematic		
large		

1 Lek et al., in press

SMALL SIMULATION STUDY[1]

True latent mean difference = .5

	PPP	estimate
unsystematic	.269	.477
systematic	.368	.789
large		

1 Lek et al., in press

SMALL SIMULATION STUDY[1]

True latent mean difference = .5

	PPP	estimate
unsystematic	.269	.477
systematic	.368	.789
large	.186	.642

1 Lek et al., in press

GOAL 2: DETECT LARGE DIFFERENCES

Problem: Large differences bias the approximate solution

solution 1: free non-invariant parameters

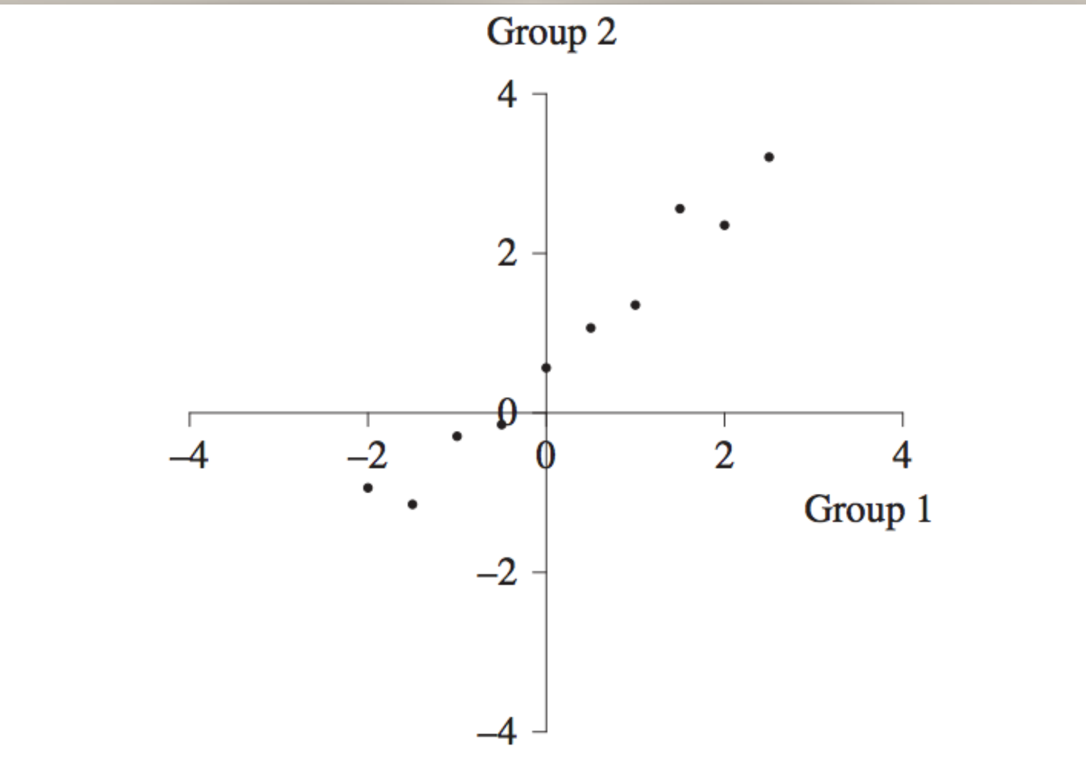
DIFFERENCE OUTPUT

			NU1_1	NU2_1
5	0.026	0.053	-0.095*	0.095*
			NU1_2	NU2_2
6	0.022	0.049	0.105*	-0.105*
			NU1_3	NU2_3
7	-0.003	0.046	-0.055	0.055
			NU1_4	NU2_4
8	0.050	0.041	0.062*	-0.062*

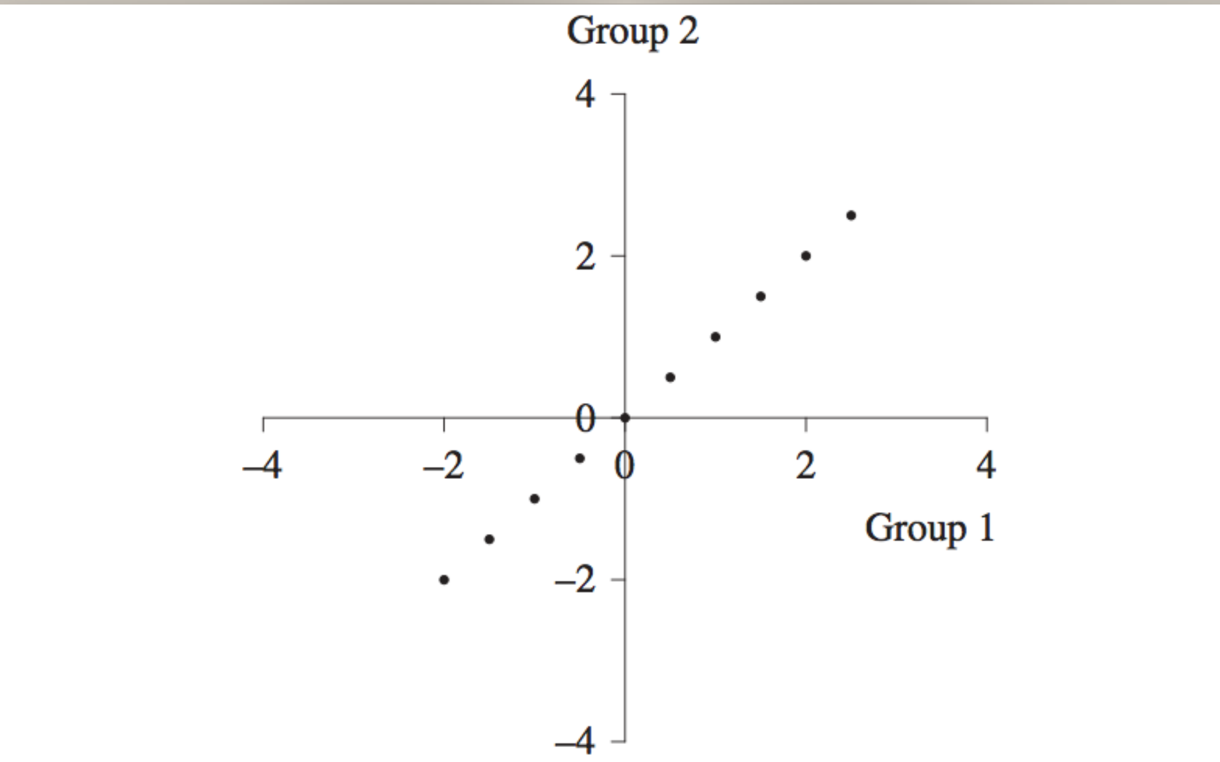
GOAL 2: DETECT LARGE DIFFERENCES

Problem: Large differences bias the approximate solution

solution 2: alignment[1]



Unaligned: Configural model (mean=0, variance=1 in both groups)



Aligned: Taking into account the group differences in means and variances

1 Asparouhov & Muthén, 2014

CONCLUSION

Bayesian approximate M.I. offers a pragmatic solution when exact M.I. is too strict in large datasets

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When there are also large differences, opt for partial Bayesian approximate M.I. or alignment

CONCLUSION

Bayesian approximate M.I. offers a pragmatic solution when exact M.I. is too strict in large datasets

Bayesian approximate M.I. works well when differences are small and unsystematic

When there are also large differences, opt for partial Bayesian approximate M.I. or alignment

More research is necessary into the role of deviating parameters and the size of the prior variance.

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APPLICATION: ACADEMIC STRESS SURROUNDING A MIDTERM

- **Sample:** 144 undergraduate students enrolled in an Intro Psych class at UC Merced[4]
- **Design:** three time points
  - T1: One week before midterm
  - T2: Right after midterm
  - T3: One week after midterm
- **Measure:** sub scale (9 items) of the Lakaev Academic Stress Response Scale (LASRS)[5]
  - Example: “I felt overwhelmed by the demands of study”
- **Research question:** Does taking a midterm affect how students assess their academic stress?

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[4] data collected by Arroyo & Winter, 2017

[5] Lakaev, 2009

DIFFERENCE PRIOR CHOICE

- Various difference priors were examined to select the appropriate variance;
- The BIC was used to select the appropriate variance
  - This strategy requires further systematic examination

$$Diff \sim N(0, \sigma^2)$$

σ^2	BIC
0.01	8882.59
0.10	8862.36
0.25	8861.24
0.50	8861.10
0.75	8861.11

BAYESIAN APPROACH

- Items with non-invariant intercepts:
  - I feel overwhelmed by the demands of study (all T)
  - There is so much going on that I can’t think straight (T1)
  - My emotions stop me from studying (T1)
  - I have trouble remembering notes (T3)
  - I felt worried about coping with my studies (T1)
  - I had difficulty eating (all T)
- Items that were invariant:
  - I felt emotionally drained by university
  - I felt emotional
  - My work built up so much that I felt like crying